

### 3.7 implicit differentiation

explicit

$$y = -x^2 + x$$

implicit

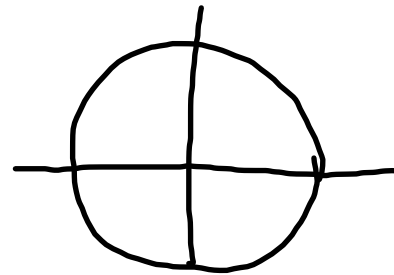
$$y + x^2 - x = 0$$

$x$ 's,  $y$ 's mixed together

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ex. implicit

$$x^2 + y^2 = 1$$



explicit  
(solve for  $y$ )

$$y = \pm \sqrt{1 - x^2}$$

$$y^4 + 2xy + y^2 = 8$$

implicit

explicit may be hard/impossible

$$y = -x^2 + x \quad \text{find } \frac{dy}{dx}$$

$$\frac{dy}{dx} = -2x + 1 \quad \text{explicit}$$

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$$y + x^2 - x = 0$$

implicit  
take der of each  
term

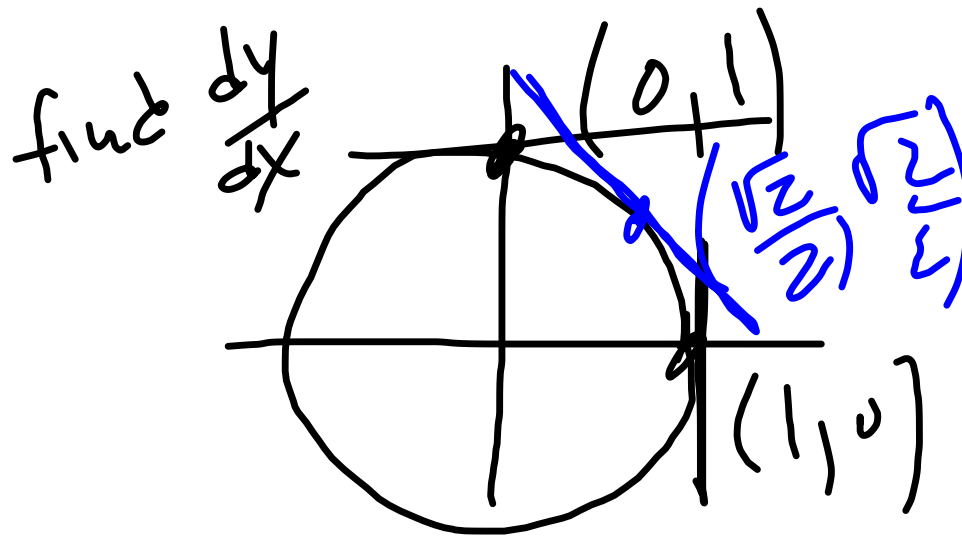
$$\frac{dy}{dx} + 2x - 1 = 0$$

solve for  $\frac{dy}{dx}$

$$\frac{dy}{dx} = -2x + 1$$

$$x^2 + \underbrace{(y)^2}_{\text{in}} = 1 \quad \text{find } \frac{dy}{dx}$$

$$2x + 2y \cdot \frac{dy}{dx} = 0$$



$$\cancel{2y} \frac{dy}{dx} = \frac{-2x}{\cancel{2x}} = -\frac{x}{y}$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

$$(y)^4 + 2xy + y^2 = 8 \quad \text{find } \frac{dy}{dx}$$

$$4y^3 \frac{dy}{dx} + (2x \cdot \frac{dy}{dx} + y \cdot 2) + 2y \frac{dy}{dx} = 0$$

$$4y^3 \frac{dy}{dx} + 2x \frac{dy}{dx} + 2y \frac{dy}{dx} = -2y$$

$$\frac{dy}{dx} (4y^3 + 2x + 2y) = -2y$$

$$\frac{dy}{dx} = \frac{-2y}{4y^3 + 2x + 2y} = \frac{-y}{2y^3 + x + y}$$

EX 5  $2x^3 - 3y^2 = 8$  find  $y''$

first get  $y'$   $6x^2 - 6yy' = 0$

$$y' = \frac{-6x^2}{-6y} = \frac{x^2}{y}$$

find  $y''$

$$y' = \frac{x^2}{y}$$

$$y'' = \frac{y \cdot 2x - x^2 \cdot y'}{y^2}$$

$$y'' = \frac{\frac{y}{y} 2xy - x^2 \frac{x^2}{y}}{y^2} = \left( \frac{2xy^2 - x^4}{y} \right) \frac{1}{y^2}$$

$$y'' = \frac{2xy^2 - x^4}{y^3}$$